Syllabus for Ph. D. (Mathematics) Entrance Examination 2015

Linear Algebra: Finite dimensional vector spaces, Linear transformations and their matrix representations, rank, systems of linear equations, eigen values and eigen vectors, minimal polynomial, Cayley-Hamilton Theroem, diagonalisation, Hermitian, Skew-Hermitian and unitary matrices, Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, self-adjoint operators.

Complex Analysis: Analytic functions, conformal mappings, bilinear transformations, complex integration, Cauchy's integral theorem and formula, Liouville's theorem, maximum modulus principle, Taylor and Laurent's series, residue theorem and applications for evaluating real integrals.

Real Analysis: Sequences and series of functions, uniform convergence, power series, Fourier series, functions of several variables, maxima, minima; Riemann integration, multiple integrals, line, surface and volume integrals, theorems of Green, Stokes and Gauss, metric spaces, completeness, Weierstrass approximation theorem, compactness, Lebesgue measure, measurable functions, Lebesgue integral, Fatou's lemma, dominated convergence theorem.

Ordinary Differential Equations: First order ordinary differential equations, existence and uniqueness theorems, systems of linear first order ordinary differential equations, linear ordinary differential equations of higher order with constant coefficients, linear second order ordinary differential equations with variable coefficients, method of Laplace transforms for solving ordinary differential equations, series solutions, Legendre and Bessel functions and their orthogonality.

Algebra: Normal subgroups and homomorphism theorems, automorphisms, Group actions, Sylow's theorems and their applications, Euclidean domains, Principle ideal domains and unique factorization domains, Prime ideals and maximal ideals in commutative rings, Fields, finite fields.

Functional Analysis: Banach spaces, Hahn-Banach extension theorem, open mapping and closed graph theorems, principle of uniform boundedness, Hilbert spaces, orthonormal bases, Riesz representation theorem, bounded linear operators.

Numerical Analysis: Numerical solution of algebraic and transcendental equations, bisection, secant method, Newton-Raphson method, fixed point iteration; interpolation, error of polynomial interpolation, Lagrange, Newton interpolations, numerical differentiation, numerical integration, Trapezoidal and Simpson rules, Gauss Legendre quadrature, method of undetermined parameters, least square polynomial approximation, numerical solution of systems of linear equations, direct methods (Gauss elimination, LU decomposition), iterative methods (Jacobi and Gauss-Seidel), matrix eigen value problems, power method, numerical solution of ordinary differential equations, initial value problems, Taylor series methods, Euler's method, Runge-Kutta methods.

Partial Differential Equations: Linear and quasilinear first order partial differential equations, method of characteristics, second order linear equations in two variables and their classification, Cauchy, Dirichlet and Neumann problems, solutions of Laplace, wave and diffusion equations in two variables, Fourier series and Fourier transform and Laplace transform methods of solutions for the above equations.

Mechanics: Virtual work, Lagrange's equations for holonomic systems, Hamiltonian equations.

Topology: Basic concepts of topology, product topology, connectedness, compactness, countability and separation axioms, Urysohn's Lemma.

Probability and Statistics: Probability space, conditional probability, Bayes theorem, independence, Random variables, joint and conditional distributions, standard probability distributions and their properties, expectation, conditional expectation, moments, Weak and strong law of large numbers, central limit theorem, Sampling distributions, UMVU estimators, maximum likelihood estimators, Testing of hypotheses, standard parametric tests based on normal, $X2$, t, F – distributions; Linear regression and correlation, Interval estimation.

Linear programming: Linear programming problem and its formulation, convex sets and their properties, graphical method, basic feasible solution, simplex method, big-M and two phase methods, infeasible and unbounded LPP's, alternate optima, Dual problem and duality theorems, dual simplex method and its application in post optimality analysis, Balanced and unbalanced transportation problems, u -v method for solving transportation problems, Hungarian method for solving assignment problems.

Calculus of Variation and Integral Equations: Variation problems with fixed boundaries, sufficient conditions for extremum, linear integral equations of Fredholm and Volterra type, their iterative solutions.